Economics *Semester 2, 2016*

Money, Banking, and Financial Markets (ECON3350)

Tutorial 4. Taylor Series, Logarithmic and Exponential Function

Solve all questions. Questions with an asterisk (*) will be marked.

Taylor Series Approximation

A k-times differentiable function can be approximated by a Taylor series:

$$f(x) \approx f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f'''(a)}{k!}(x-a)^k$$

Here, $f(a)$ is the function value at $x = a$, $f'(a)$ is the rate of change at $x = a$, etc..

Question 1*

a) Determine the function value and the first and second derivative of $f(x) = x^4$ at $x = 3$. Then, estimate the function value at $x = 3.1$, using a first-order and second-order Taylor series approximation. How big is the error? Illustrate with a graph.

b) Determine the function value and the first and second derivative of $f(x) = e^x$ at $x = 0$. Then, estimate the function value at $x = 0.1$, using a first-order and second-order Taylor series approximation. How big is the error? Illustrate with a graph.

Question 2

Show that the change in the logarithm of a variable is approximately equal to a percentage change.

Hint: Use a first-order Taylor approximation of the logarithmic function.

Question 3

Show that $\ln(1+R) \approx R$ for small R.

Hint: Use a first-order Taylor approximation around $R = 1$.

(This approximation is often used in financial economics. For an interest rate of 3%, the logarithm of the gross return (1+R) is approximately equal to R: $\ln(1+0.03) \approx 0.03.$)

Question 4*

Show that the time derivative of the logarithm of a variable equals its growth rate measured in percent. Use the following notation for a time derivative: $dX/dt \equiv \dot{X}$.

Question 5

Use the fact that the time derivative of the logarithm of a variable equals its growth rate to show that

a) The growth rate of the product of two variables, $X(t)Y(t)$, equals the sum of their growth rates.

b) The growth rate of the ratio of two variables, $X(t)/Y(t)$, equals the difference of their growth rates.

c) If $Z(t) = X(t)^{\alpha}$, then $\dot{Z}(t)/Z(t) = \alpha\dot{X}(t)/X(t)$.

Question 6*

a) The exponential function e^x is the inverse of the logarithmic function $\ln x$. Illustrate with a graph.

b) Does every function have an inverse?